

Mott transition and integrable lattice models in two dimensions

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(Dated: December 9, 2008)

We describe the two-dimensional Mott transition in a Hubbard-like model with nearest neighbors interactions based on a recent solution to the Zamolodchikov tetrahedron equation, which extends the notion of integrability to two-dimensional lattice systems. At the Mott transition, we find that the system is in a d -density wave or staggered flux phase that can be described by a double Chern Simons effective theory with symmetry $su(2)_1 \otimes \widehat{su(2)}_1$. The Mott transition is of topological nature, characterized by the emergence of vortices in antiferromagnetic arrays interacting strongly with the electric charges and an electric-magnetic duality. We also consider the effect of small doping on this theory and show that it leads to a quantum gas-liquid coexistence phase, which belongs to the Ising universality class and which is consistent with several experimental observations.

In spite of substantial advances in our theoretical understanding of strongly correlated electron systems, several problems still continue to provide stimulating challenges. One of the most interesting among these is the Mott transition, or metal-insulator transition driven by correlations. As early as in 1939, Mott argued that if the electron density in a metallic system was lowered enough, the Coulomb repulsion would dominate over the kinetic energy so that the system would undergo a transition to an insulating regime [1]. From the experimental point of view, there exist several systems which display a Mott-type transition, such as vanadium oxide V_2O_3 , several organic conductors, some doped semiconductors, and even underdoped high T_C superconductors. Moreover, coexistence between phases of different densities has been observed in several experiments [3]. From the theoretical point of view, finding solutions to even the simplest models (such as the Hubbard model) is difficult, given the failure of perturbative approaches due to the narrow differences separating the localized regime of the electrons in the insulating phase and the itinerant one in the conducting state.

As of today, there exist two basic approaches for studying this transition: one is the dynamical mean field theory (DMFT) method[2], valid in the limit of infinite dimensions (or infinite coordination number), which maps the Hubbard model onto the impurity Anderson model, with the addition of a self-consistency condition. This framework neglects spatial correlations while retaining the on-site quantum ones. The second approach consists in finding analytic expressions for physical observables in integrable models exhibiting the Mott behavior, for example, by using the Bethe Ansatz or the bosonization methods [4] [5] [6]. However, the main restriction of these models is that they are formulated in one spatial dimension, unlike most system of experimental interest. The

goal of the present article is to extend this approach to a two-dimensional (integrable) lattice model that exhibits the Mott transition and write down an Effective field Theory to further analyze the behavior of the model.

Let us start by consider a system of spinless fermions on a square (two-dimensional) lattice with hamiltonian:

$$H = -\frac{t}{2} \sum_{i,\mu} [\psi^\dagger(i + e_\mu) e^{iA_\mu} \psi(i) + h.c.] + U \sum_{i,\mu} \rho(i) \rho(i + e_\mu), \quad (1)$$

where i labels the lattice sites and e_μ are unit lattice vectors, t is the hopping parameter, U is the (constant) Coulomb potential, $\rho(i)$ is the normal ordered charge density with respect to the half-filling ground state, $\rho(i) =: \psi_i^\dagger \psi_i : - 1/2$ and A_μ is the abelian statistical gauge field, which after imposing Gauss' law constraint reads

$$A_\mu(i) = \sum_k [\Theta(k, i) - \Theta(k, i + e_\mu)] \psi_k^\dagger \psi_k, \quad (2)$$

where $\Theta(k, i)$ is the angle between the chosen direction i and an arbitrary one k on the lattice. Note that, for the one-dimensional case, the gauge field is irrelevant, in agreement with the fact that quantum statistics in one spatial dimension is arbitrary [7] and does not involve any physical gauge field. Using the two-dimensional Jordan-Wigner transformation[9][7] :

$$\begin{aligned} S_j^+ &= \psi_j^\dagger U_{2d}(j) \\ S_j^- &= U_{2d}(j) \psi_j \\ S_j^z &= \psi_j^\dagger \psi_j - \frac{1}{2}, \end{aligned} \quad (3)$$

where $U_{2d}(j) = \exp[i \sum_{k \neq j} \Theta(k, j) \psi_k^\dagger \psi_k]$, the hamiltonian

nian (1) becomes that of a XXZ Heisenberg model

$$H_{XXZ} = \sum_{\langle i,j \rangle} [-(S_j^x S_j^x + S_i^y S_j^y) + \Delta S_j^z S_j^z], \quad (4)$$

where we have rescaled the terms such that $\Delta = U/t$. Following [8] we define an *interaction star* as the set of points where the spins entering in an elementary interaction are localized, *i.e.*, the central site and their nearest-neighbors in the XXZ model. The n -th interaction star has an energy $E_{XXZ}([\sigma]_n)$ which depends on the spin configuration in the star and on the local Boltzmann weights $W([\sigma]_n)$. Therefore, the partition function takes the form $Z = \sum_{\sigma} \prod_n W([\sigma]_n)$, where the sum is taken over all possible configurations of the entire lattice.

In two dimensional quantum systems and three dimensional statistical models the integrability is guaranteed by the existence of a set of mutual commuting layer-to-layer transfer matrices $T_{mn}(l, \mu)$, which is tantamount to the existence of solutions of the so-called Zamolodchikov's tetrahedron equation (TE) [10][11]:

$$R_{abc} R_{ade} R_{bdf} R_{cef} = R_{cef} R_{bdf} R_{adc} R_{abc}, \quad (5)$$

where the operators R_{ijk} define the mapping $R_{ijk} : V_i \otimes V_j \otimes V_k \rightarrow V_i \otimes V_j \otimes V_k$, and V_n is the spin one-half representation space, such that their matrix elements are the Boltzmann weights of the vertex $R_{ijk} = W([\sigma]_{ijk})$ (the indices i, j, k label the interaction star). These can be rewritten as the $LLLR - RLLL$ operator conditions, which express the associativity of the Zamolodchikov algebra:

$$L_{12,a} L_{13,b} L_{23,c} R_{abc} = R_{abc} L_{23,c} L_{13,b} L_{12,a}, \quad (6)$$

where, for example, the operator $L_{12,a}$ acts on $V_1 \otimes V_2 \otimes F_a$, V_1, V_2 are the auxiliary spaces and F_a is the quantum space. If F_a is the representation space of some algebra \mathcal{A} , it is possible to interpret the operators $L_{ij,a}$ as operator-valued matrices acting on $V_1 \otimes V_2$, and depending 'parametrically' on the generators of the algebra \mathcal{A} denoted by v_a and, possibly, on some c -numbers denoted by s_a : $L_{12,a} = L_{12}(v_a, s_a)$. In this case, the equation (6) can be expressed as a 'local Yang Baxter' equation:

$$L_{12}(\mathbf{v}_a, s_a) L_{13}(\mathbf{v}_b, s_b) L_{23}(\mathbf{v}_c, s_c) = L_{23}(\mathbf{v}'_c, s_c) L_{13}(\mathbf{v}'_b, s_b) L_{12}(\mathbf{v}'_a, s_a). \quad (7)$$

The tetrahedron equation (5) is highly non-trivial to solve, but recently a new solution to it has been found in [11]. The solution is associated to the finite-dimensional highest-weight representations of the quantum affine algebras $U_q(\widehat{sl}(2))$, displaying the three-dimensional structure of quantum groups. It may be understood as a quantization of the spatial fluctuations of geometrical extended objects, and we shall see that in our case that

these may be reinterpreted as (discrete) charge density waves. For completeness, we now briefly review the new solution (for details see [11]). The solution is inspired in the geometry of transformations applied to an hexahedron (see fig.(1): there are three independent angles on each face and nine angles to fix the spatial orientation of the hexahedron. Therefore, nine independent angles are needed to specify it. Let us consider the mapping,

$$R_{123} : [\alpha_j, \beta_j, \gamma_j] \rightarrow [\alpha'_j, \beta'_j, \gamma'_j], \quad (8)$$

where α, β and γ are the angles of the j -th face, and the primed variables refer to the opposite faces. For each

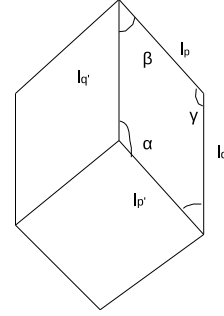


FIG. 1: Diagram of a hexahedron in which the angles (α, β, γ) and side lengths (l_p, l'_p, l_q, l'_q) for one face are indicated.

quadrilateral face, say the $1'$, the relationship between opposite sides is given by $(l'_p, l'_q)^t = X(\mathcal{A}_1)(l_p, l_q)^t$, where $X(\mathcal{A}_1)$ is a matrix acting non trivially on the face $1'$, that depends on the angles on that face ($\mathcal{A}_1 = (\alpha_1, \beta_1, \gamma_1)$), and which for a circular lattice reads:

$$X(\mathcal{A}_1) = \begin{bmatrix} k_1 & a_1^* & 0 \\ -a_1 & k_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where $k_1 = \cos \alpha_1 \sin \beta_1$, $a = \cos \alpha_1 \sin(\alpha_1 + \beta_1)$ and $a_1^* = \cos \alpha_1 \sin(\alpha_1 - \beta_1)$. In the general case, considering three faces, we have:

$$(l'_p, l'_q, l'_r)^t = X_{pq}(\mathcal{A}_1) X_{qr}(\mathcal{A}_2) X_{rs}(\mathcal{A}_3) (l_p, l_q, l_r)^t. \quad (9)$$

The same result (9) is obtained by using the opposite faces with angles $(\mathcal{A}'_1, \mathcal{A}'_2, \mathcal{A}'_3)$. Therefore, it is easy to see that there exists a functional mapping given by $X_{pq}(\mathcal{A}_1) X_{qr}(\mathcal{A}_2) X_{rs}(\mathcal{A}_3) = R_{123}(X_{rs}(\mathcal{A}_3) X_{qr}(\mathcal{A}_2) X_{pq}(\mathcal{A}_1))$. This relation could be considered as a 'gauge symmetry'. It can be shown that the mapping R_{123} satisfies the functional tetrahedron equation (FTE), a 'classical version' of the tetrahedron equation:

$$\begin{aligned} k'_2 a_1^* &= k_3 a_1^* - k_1 a_2^* a_3 & k'_2 a'_1 &= k_3 a_1 - k_1 a_2^* a_3^* \\ a_2^* &= a_2^* a_3^* + k_1 k_3 a_2^* & a'_2 &= a_2 a_3 + k_1 k_3 a_2 \\ k'_2 a_3^* &= k_1 a_3^* - k_3 a_1 a_2^* & k'_2 a'_3 &= k_1 a_3 - k_3 a_1^* a_2, \end{aligned}$$

with $k' = \sqrt{1 - a_2 a_2^*}$. This map defines a canonical transformation of the Poisson algebra, which in terms of the angles reads: $\{\alpha_i \beta_j\} = \delta_{ij}$ $\{\alpha_i, \alpha_j\} = 0$ $\{\beta_i, \beta_j\} = 0$. We now canonically quantize this theory, *i.e.*, by replacing the angles by Hilbert space operators and the Poisson brackets by commutators, so that $[\alpha, \beta] = \zeta \hbar$, where ζ is a complex parameter. It can be shown that the quantum operators corresponding to k, a, a^* , satisfy the commutation relation of the q -oscillator algebra

$$\begin{aligned} qa^\dagger a - q^{-1}aa^\dagger &= q - q^{-1} \\ ka^\dagger &= qa^\dagger k \quad ka = q^{-1}ak, \end{aligned} \quad (10)$$

with quantum deformation parameter $q = e^{\zeta \hbar}$ and $k^2 = q(1 - a^\dagger a)$. Upon this quantization, the map R_{123} becomes a quantum operator satisfying by construction the quantum tetrahedron equation (5). It has been shown in [11] that it is possible to construct the matrix elements $\langle n'_1, n'_2, n'_3 | R | n_1, n_2, n_3 \rangle$ in the basis of the Fock space constructed from the q -oscillator algebra. The operators R_{ijk} define an automorphism of the triplets of tensor products of the q -oscillator algebra $O_q^{\otimes 3} \rightarrow O_q^{\otimes 3}$, and it also has the property of being non-degenerate in $F^{\otimes 3}$. This fact, together with the tetrahedron equation (5), implies the validity of the standard Yang-Baxter equation (which signals the integrability in two dimensions). In fact, tracing out in the Fock space F_a , the following equations are obtained:

$$R_{bc}R_{bd}R_{cd} = R_{cd}R_{bd}R_{bc} \quad (11)$$

$$L_{Vb}L_{Vc}R_{bc} = R_{bc}L_{Vc}L_{Vb}. \quad (12)$$

One affinization of the solution of [11] has been given in [12] as follows: consider the layer to layer transfer matrix $T_{mn}(l, \mu)$ which are related to the L -operators by $T_{mn}(l, \mu) = \prod_{i=1}^n \prod_{j=m}^1 L_j(l_i \mu_j)$ and may be obtained from an ansatz that solves the local Yang-Baxter equation:

$$L_{1,2}(u_3, l_3) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & l_3 k_3 & a_3^\dagger & 0 \\ 0 & -q^{-1}l_3 \mu a_3 & \mu_3 k_3 & 0 \\ 0 & 0 & 0 & -q^{-1}l_3 \mu_3 \end{bmatrix}$$

where we have chosen the Fock space F_a as F_3 , for convenience. It has been shown that the solution of the affine TE (when all parameters l_i are equal to each other) has symmetry $U_q(\widehat{sl(n)})$, where n is an arbitrary integer and can be considered as the emergent coordinate of the third dimension.

Equations (11)(12) can be associated either to an integrable two-dimensional statistical system or to a quantum system in $(1+1)$ dimensions. Therefore, the new solution of the Zamolodchikov equation tells us that is possible to break up the $(2+1)$ -dimensional lattice system (in a consistent fashion) into $(1+1)$ -dimensional ones

for each row and column of the lattice or into classical statistical systems in $(2+0)$ -dimensions at a fixed time. This result relates the one-dimensional Mott point (*i.e.*, the critical value of t/U in the one-dimensional analogue of (1) and its order parameter) to its two-dimensional counterpart as follows: on the one hand, in each row or column the XXZ spin system with hamiltonian,

$$H_{XXZ}^d = - \sum_{i=1}^L (S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta S_i^z S_{i+1}^z) + H_b \quad (13)$$

where $H_b = \alpha(S_1^z - S_L^z)$ and $\Delta = (q + q^{-1})/2$ and $\alpha = (q - q^{-1})/2$, is known to possess the symmetry $U_q(sl(2)) \otimes U_q(sl(2))$ [13]. Note that $q = 1$ at $\Delta = 1$, so that the boundary conditions are irrelevant, and we can choose periodic boundary conditions without breaking the quantum group symmetry. The system defined by (13) can be mapped to a one-dimensional nearest neighbors Hubbard (fermionic) system (using a Jordan-Wigner transformation) and may also be bosonized as a Luttinger system, which is a conformal field theory (CFT) [24] with central charge $c = 1$ [15][16]. It has been shown that this system undergoes a Mott transition at $\Delta = 1$ [6], becoming an insulator. Moreover, at the Mott transition (with $q = 1$), the quantum group symmetry becomes the ordinary $su(2)$ symmetry (as far as the algebraic and symmetry properties are concerned, we treat $su(2)$ and $sl(2)$ as interchangeable) and the CFT will have symmetry $\widehat{su(2)}_1 \otimes \widehat{su(2)}_1$ (because two CFTs, one for each chirality are needed) which becomes $su(2) \otimes su(2)$ at long distances. This CFT can be realized by a Wess-Zumino-Witten (WZW) model at level $k = 1$, or by two chiral bosons (of opposite chirality) compactified on a circle at the self-dual radius (under R -duality), where the vertex operators, known as currents $J^\pm = e^{i\sqrt{2}\phi}$, are well defined under a shift in the zero-mode of the fundamental bosonic field of the WZW model $\phi \rightarrow \phi + 2\pi r$ (r is the compactification radius). The currents have scaling dimension $(1, 0)$ and together with $J^3(z) = i\partial\phi$, of the same dimension, satisfy the $\widehat{su(2)}_1$ algebra. The R -duality in these systems is well-known ([24]), for which an exchange of r for $1/r$ leaves the spectrum unchanged but the elementary degrees of freedom are exchanged between charges and vortices.

For $\Delta = 1 + \epsilon$, the system (13) develops an energy gap in the spectrum $E_g = 4 \exp(a/\epsilon)$ and exhibits a charge density wave (CDW) order parameter [6], defined by $\langle \rho(i) \rangle = 1/2[1 + (-1)^i P]$, where $P = \langle \psi_r^\dagger(i) \psi_l(i) + \psi_l^\dagger(i) \psi_r(i) \rangle = 1/\sqrt{\epsilon} \exp(-a/\sqrt{\epsilon})$ (a is a constant, i labels the lattice site and the expectation value is taken in the ground state). The corresponding low-energy effective theory is given by the Sine-Gordon theory for the non-chiral effective bosonic field ϕ , of Lagrangian density:

$$L_{SG} = \frac{1}{2}(\partial\phi)^2 + \beta \cos \sqrt{2}\phi. \quad (14)$$

The last term can be interpreted as having origin in the Umklap processes naturally arising in the lattice fermionic description, in which it is given by $\langle \psi_L \psi_L \psi_R \psi_R \rangle$ (where $\psi_L = \exp(-i\phi_L/\sqrt{2})$ and $\psi_R = \exp(i\phi_R/\sqrt{2})$, with $\phi = \phi_L + \phi_R$). As it has been pointed out in [17], for $\beta > 0$ (repulsive interactions), the ground state energy is minimized for a constant ground state density $\langle \phi \rangle = \pi/\sqrt{2}$, which in the fermionic representation corresponds to $\langle \psi_R^\dagger \psi_L \rangle = -\langle \psi_R^\dagger \psi_L \rangle = if$, where f is a real function that changes sign under a $\pi/2$ rotation around any axis (with appropriate generalizations for the one-dimensional case [17]). Therefore, this one-dimensional system exhibits a d -wave density order parameter, meaning that on the ground state the quantity $\langle \psi^\dagger(k) \psi(k + \pi/a) \rangle$ breaks individually time reversal, translation invariance by one lattice site and $\pi/2$ rotation (around any axis) symmetries, while preserving the composition of any two of them. (Note that $\langle \psi_R^\dagger \psi_L \rangle = -\langle \psi_R^\dagger \psi_L \rangle$ yields $P = 0$, implying a constant density profile.

As consequence of the projection-like character of the solution of [11], by consistency, the system on the lattice must be in a (two-dimensional) d -density wave and also will be described by a CFT with $c = 1$ in all phases. In order to further account for the charge neutrality of the spin excitations and the additional lattice symmetries [34], the CFT should be taken on the orbifold S^1/Z_2 and modded out by the lattice symmetry D_4 . This CFT has been identified as characterizing the critical point of the six-vertex model or the four-state Potts model [18].

On the other hand, the lattice fermion system can be viewed in an alternative way: consider a two-layered system periodic in the 'temporal axis', and trace out over the temporal dimension. The resulting Yang-Baxter equation has symmetry $U_q(\widehat{sl(2)}) \otimes U_q(\widehat{sl(2)})$ corresponding to the symmetry of the six-vertex model [13], whose phase diagram depends on the Boltzmann weights at each vertex of the lattice through the parameter $\Delta_{6v} = (a^2 + b^2 - 2c^2)/2ab$, where $a = \exp(-\beta E_a)$, $b = \exp(-\beta E_b)$, and $c = \exp(-\beta E_c)$ are the weights at each vertex. The transfer matrix of the six-vertex model is given by that of the XXZ model through a Wick rotation. At the Mott critical point this implies that ($\Delta = -\Delta_{6v} = 1$) it is in the antiferroelectric phase. (Note that the spins are also rotated changing the sign of the spin-wave term [15])

Now we would like to write down an effective field theory (EFT) for the model on the square lattice at the Mott critical point [19]. We first choose the effective degrees of freedom, and impose their characteristic symmetries on the theory under construction. In our case, both symmetries are given by the exact solution discussed above. As it was first pointed out by Witten, there is a close relationship between quantum groups, vertex models and Chern-Simons (CS) gauge theories: the expectation values of the Wilson loops can be calculated as statisti-

cal sums of Boltzmann weights in suitable defined vertex models, so that the mathematical structure of quantum groups encodes the topology of planar Wilson loops [22]. However, CS theories possess naturally the symmetry $U_q(\widehat{sl(2)})$ (with $q = \exp(2\pi i/k)$ where k is the CS coupling constant) [26] [27], and not $U_q(\widehat{sl(2)}) \otimes U_q(\widehat{sl(2)})$. This mismatch is a consequence of the absence of parity conservation in the CS gauge theories. The simplest CS-type theories that preserve parity are the double CS gauge theories (which contain two $u(1)$ chiral gauge fields of opposite chirality, namely right and left)[20]:

$$S_{DCS} = \frac{k}{4\pi} \int d^3x a_R \wedge da_R - \frac{k}{4\pi} \int d^3x a_L \wedge da_L \quad (15)$$

where a_R (a_L) denotes the right (left) gauge field. This theory is known to be equivalent to the BF theory [21] and it can also be written as a mixed CS theory. Here we are using $a \wedge da$ as a short-hand notation for the lattice version of the CS coupling $a_\mu K_{\mu,\nu} a_\nu$ with $K_{\mu,\nu} = S_{\mu\nu} \epsilon_{\mu,\alpha,\nu} d_\alpha$, $S_\mu f(x) = f(x + a\epsilon_\mu)$, $S_\mu f(x) = (f(x + a\epsilon_\mu) - f(x))/a$, (with a the lattice spacing). At the Mott transition we have $k = 1$, since the coupling k fixes the unit of charge and the statistics of the excitations, and we find consistency with the fact that the effective degrees of freedom are density waves of the underlying electron system. Note that the $u(1)$ CS theory can be considered as the broken parity phase of the $su(2)$ CS theory, where the relation to the six-vertex model has been established [22] [23].

We now impose periodic boundary conditions to the EFT, *i.e.*, compactify the space domain on a torus. Cutting down the torus along any cycle, induces the loosening of gauge symmetry on the cycle, so that the gauge fields become boundary dynamical degrees of freedom [29][31][30] which are free chiral bosons ($c = 1$ CFTs), representing charge density waves (CDW) described also by Luttinger systems. In the quantum theory obtained after quantizing these classical bosonic waves, there is a shift in the coupling parameter k that is properly taken into account by the Sugawara construction: $k \rightarrow k + c_v$, where c_v is the dual coxeter number of the symmetry algebra of the gauge group ($c_v = 2$ for $su(2)$). However, the identification of the Mott transition is done at the classical level, implying that the topological order remains given by the relation $q = \exp(2\pi i/k)$, with no shift in k .

We would now like to show that in the EFT, the emergence of a d -wave order considered before is natural. For this, we focus on the electric-magnetic duality between charges and vortices implicit in the EFT. Let us consider the charge current $j^\mu(x)$ degrees of freedom in the direct lattice, and corresponding vortex current $\phi^\mu(X_d)$ in the dual lattice (whose sites are in the center on each plaquette of the direct lattice). We assume that these degrees of freedom can couple, and the low-energy action for their

interaction is given by a mixed Chern Simons theory

$$S_{MCS} = \frac{k}{4\pi} \int d^3x a \wedge da - \frac{k}{4\pi} \int d^3x b \wedge db + a_\mu \phi_\mu \quad (16)$$

where we have introduced two gauge fields a_μ and b_μ for the current and vortex degrees of freedom, respectively. The relevant definitions are $j^\mu = k\hat{K}_{\mu,\nu}a_\nu$, $\phi^\mu = k\hat{K}_{\mu,\nu}b_\nu$, $\hat{K}_{\mu,\nu} = \hat{S}_{mu}\epsilon_{\mu,\alpha,\nu}\hat{d}_\alpha$, $\hat{S}_\mu f(x) = f(x + a\epsilon_\mu)$, $\hat{d}_\mu f(x) = (f(x - a\epsilon_\mu) - f(x))/a$. In the dual lattice, the system dual to the original one is a two-dimensional XXZ spin system with coupling constant Δ^{-1} . The degrees of freedom corresponding to these spin variables are vortices. For $\Delta < 1$ the spins in the dual system are frozen, there are no spin waves, and the system is in the antiferromagnetic phase. Therefore, its effective action is given by a CS theory with punctures[14]:

$$S_{CS} = \frac{k}{4\pi} \int d^3x a^\mu K_{\mu,\nu} a_\nu + \sum_p \phi^0 [\delta(x_d, y_d) - \delta(x_d + 1, y) - \delta(x_d, y_d + 1) + \delta(x_d + 1, y_d + 1)] ,$$

where a_λ is a (different) abelian CS field and \sum_p means that the sum is taken over all fundamental domains. Each domain has period $2a$ and contains four vortices in antiferromagnetic array. Therefore, the classical low-lying states reproduce an antiferromagnetic current pattern. At the quantum level, Gauss' law selects the physical states from the lattice CS gauge theory with punctures. The quantum order of the ground state of this theory (staggered flux phase) breaks translation and parity symmetries by one lattice site and also time reversal invariance, but it is, however, invariant under the composition of any two symmetry transformations, satisfying the definition of the d -density wave order invariance ([17]). Therefore, the EFT analysis shows that the Mott transition for the system defined by (1) is of topological (*i.e.*, Kosterlitz-Thoulose type) nature, characterized by the emergence of CS vortices in antiferromagnetic arrays. Note that the resulting theory breaks the Z_2 vortex symmetry associated and, therefore, the two-dimensional chirality on each plaquette (15).

Finally, we would like to discuss the behavior of the previously considered EFT away from the Mott critical point. By analogy with the CS theory of the quantum Hall effect, we could expect a ground state stable against small doping. In that case, for the simplest Laughlin inverse filling fractions $k = m$ (m odd integer), the ground state is a droplet of incompressible quantum liquid [32] (however, other phases with more exotic quantum orders, like Nematic phases are also possible in other regimes) and is stable under small perturbations away from the center of a given plateau in the conductivity. In the Mott system, we have already assumed that the dynamically generated vortices act as external statistical fields for the new electrons injected in the system by doping (this can

be considered as an extension of the R -duality). At the self-dual point, statistical magnetic fields can be interchanged with statistical electric fields (on a torus). After imposing the lattice symmetries, the low-lying effective Hamiltonian for the injected electrons (in first quantization) is:

$$H = \sum_i [-\hbar^2 \frac{1}{2m} (\frac{\partial^2}{\partial x_i^2} + \frac{\partial^2}{\partial y_i^2}) + \lambda_i (x_i^2 - y_i^2)] , \quad (17)$$

where λ_i can take the values $\pm\lambda$. Therefore, the electric potential changes sign in $x = \pm y$, producing domain walls between regions with different electron densities. Similar results can be obtained using the W_4 symmetry, which is related to the relevant perturbations of the Ashkin-Teller and six-vertex models away from the critical point [33][35].

One consequence of having discussed the EFT is that, *a-posteriori*, the behavior of the electrons can be more easily understood. It can be shown that the interaction term in the hamiltonian (1) in the the continuum limit contains a chemical potential term of the form $-\mu \rho$, with $\mu = \Delta$, which ensures the half-filling condition. Therefore, changing the chemical potencial by doping in $\delta\mu$ modifies the hamiltonian (in the spin representation (4) by $H(\Delta) \rightarrow H(\Delta) + \delta\mu \sum_{\langle ij \rangle} S_i^z S_j^z$. At $\Delta = 1$, the dynamics of the electron system is given by the double CS theory (15), whose hamiltonian can be defined as the temporal component of the stress-energy tensor $H_{cs} = T_{00}$, where $T_{\mu\nu} = \delta S_{cs} / \delta g_{\mu\nu}$ and where $g_{\mu\nu}$ is the metric tensor. However, the CS action is topological and, therefore, independent of the metric and $H_{cs} = 0$ for each chiral component leading to $H(\Delta = 1) = 0$. This means that doping the system away from the critical point, the dynamics is controlled by an effective Ising hamiltonian.

To sum up, our study of the EFT for the model (1) shows that the Mott transition is of topological (*i.e.*, Kosterlitz-Thoulose type) nature, characterized by the emergence of CS vortices in antiferromagnetic arrays. The symmetry $\widehat{su(2)}_1 \otimes \widehat{su(2)}_1$ of the critical point implies that electric and magnetic vortices can be interchanged, providing effective attractive and repulsive forces. Doping the system produces domain walls, signaling a quantum gas-liquid phase coexistence, which belongs to the Ising universality class.

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